# 1.12 Complex numbers introduction\_P\_1

**1a.** *[2 marks]*

Let , where .

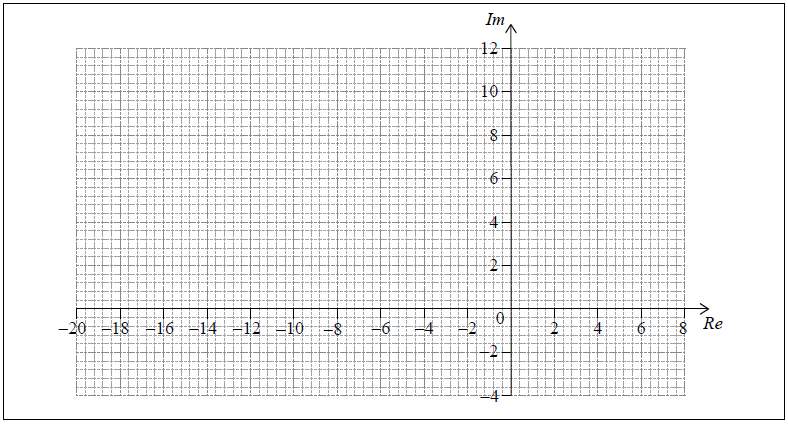
for  = 2,

find the values of , , and .



**1b.** *[3 marks]*

draw , , , and  on the following Argand diagram.



**1c.** *[2 marks]*

Let .

Find the value of  for which successive powers of  lie on a circle.



**2a.** *[5 marks]*

Find the roots of  which satisfy the condition , expressing your answers in the form , where , .



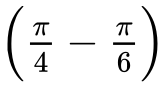
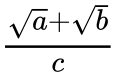
**2b.** *[4 marks]*

Let *S* be the sum of the roots found in part (a).

Show that Re *S* = Im *S*.



**2c.** *[3 marks]*

By writing  as , find the value of cos  in the form , where ,  and  are integers to be determined.



**2d.** *[4 marks]*

Hence, or otherwise, show that *S* = .



**3a.** *[4 marks]*

Consider the distinct complex numbers , where .

Find the real part of .



**3b.** *[2 marks]*

Find the value of the real part of  when .



**4a.** *[4 marks]*

Let  be one of the non-real solutions of the equation .

Determine the value of

(i)     ;

(ii)     .



**4b.** *[4 marks]*

Show that .



**4c.** *[5 marks]*

Consider the complex numbers  and , where .

Find the values of  that satisfy the equation .



**4d.** *[6 marks]*

Solve the inequality .

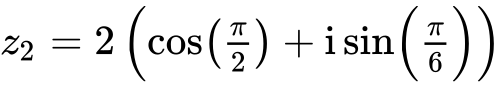


**5a.** *[6 marks]*

Solve the equation  giving your answers in the form  **and** in the form  where .



**5b.** *[11 marks]*

Consider the complex numbers  and .

(i)     Write  in the form .

(ii)     Calculate  and write in the form  where .

(iii)     Hence find the value of  in the form , where .

(iv)     Find the smallest value  such that  is a positive real number.



**6.** *[17 marks]*

A geometric sequence , with complex terms, is defined by  and .

(a)     Find the fourth term of the sequence, giving your answer in the form .

(b)     Find the sum of the first 20 terms of , giving your answer in the form  where  and  are to be determined.

A second sequence  is defined by .

(c)     (i)     Show that  is a geometric sequence.

          (ii)     State the first term.

          (iii)     Show that the common ratio is independent of *k*.

A third sequence  is defined by .

(d)     (i)     Show that  is a geometric sequence.

          (ii)     State the geometrical significance of this result with reference to points on the complex plane.

**7.** *[7 marks]*

Consider the complex numbers  and .

(a)     Given that , express *w* in the form .

(b)     Find \* and express it in the form .

Printed for SANSKAR SCHOOL

© International Baccalaureate Organization 2019

International Baccalaureate® - Baccalauréat International® - Bachillerato Internacional®